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Inductive reasoning: Problems, methods of justification and interaction between mathematics and computer science.

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ABSTRACT

Our paper reports on the foundations of inductive reasoning, more precisely, the reasons for the conflict between several scientists around the place, the notion of induction and its role in different sciences, in particular mathematics. The study starts from an epistemological and didactic reflection of this reasoning to understand this concept. First, the epistemological problems of induction in relation to its meaning and validity are presented. These problems highlight the complexity of induction. This leads us, in a second step, to set up a typology of the notion of induction developed by philosophers, linguists, empiricists and mathematicians. Finally, the researchers present a didactic analysis of mathematical induction which highlights the specific comprehension difficulties of learners and emphasizes the interaction between mathematics and computer science, in particular the relationship between the concepts of mathematical induction and recursion.

1. Introduction

The inference practiced by thought is generally divided into two main forms, one of which is deduction and the other is induction, each of the deductive and inductive reasoning has its own method and justification.

In the literature, inductive reasoning has a significant place in all sciences. Nevertheless, some empiricists, philosophers, sociologists and mathematicians mention that deductive reasoning is rigorous. On the other hand, the rigor of inductive reasoning was never justified

History shows that inductive reasoning has been one of the areas of mathematics where its links with other scientific disciplines are most apparent. Inductive reasoning is a concept that promotes interdisciplinary between mathematics and computer science.

Which begs the question: what are the reasons for the conflict between several scientists over the justification of the concept of induction and its role in the various sciences?

In our work, the authors propose to describe the devolution of inductive reasoning learning in general and by mathematical induction in particular by presenting the interaction between mathematics and computer science. In a first part an epistemological point of view on inductive and mathematical induction reasoning is presented. In the second part, the role of inductive reasoning in the typology of sciences will be studied and in the last part, the focus will be on the interaction between mathematics and computer science.

2. Epistemological problems of induction

To start with, deduction is a general-to-individual reasoning (Duquesne, 2003; Oléron, 1977). This is a reasoning in which the premises necessarily imply the conclusion (Jeannotte, 2015).

This study will illustrate two examples requiring deductive reasoning in the first, more well-known and purely logical one: *Every man*

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is mortal. Socrates is a man. Therefore Socrates is mortal.

The second example is purely mathematical*: For all natural n,* 1+2+3+…+n $= \frac{n(n+1)}{2}$, *Then* 1+2+…+100 = $\frac{100.101}{2}$ = 5050.

As far as induction is concerned, this reasoning concerns the passage from the particular to the general. Arnaud and Nicole (1965) define induction as «*when the investigation of several particular things leads us to the knowledge of a general truth*».

Cozic (2009) noted that: «*The central characteristic of inductive reasoning: its premises do not logically imply its conclusion and there is, as is sometimes said, more in the conclusion than in the premises.* »

This is demonstrated in the two examples below, which require inductive reasoning:

The first example was cited by Arnaud and Nicole (1965): «*When it has been proven on many seas that the water is salty, and on many rivers that the water is fresh, we generally conclude that sea water is salty, and river water fresh*». This example illustrates a form of inductive reasoning, *enumerative* induction, by which a universal statement is inferred.

Like Aristotle, Arnauld and Nicole show that all universal knowledge is used to know singular facts.

The second example concerns a conjecture about the number of sides of the nth shape of a "Von Koch flake", based on the first four shapes given. This example illustrates a form of inductive reasoning: *generalization* as a form of inductive reasoning.

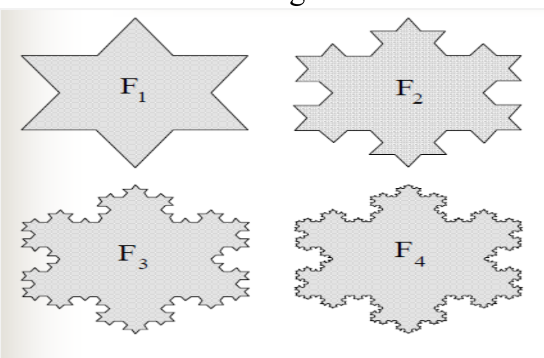

Figure1: The flake of Von Koch

From the particular cases one can conjecture a generalization: For any natural non-zero integer, the number of sides of the nth form $F_n = 3.\ 4^n$.

To conclude, the inference method in the inductive evidence is not explained and is no longer justified (a deviation from its logical construction)

Induction is devoid of any validity; the rigor of this reasoning could never be justified (uncertain) (Hume, 1871; Popper, 1973).

Inductive reasoning is a research method for different sciences. This reasoning is the passage from the particular to the general by posing new truths, but without certainty (Bernard, 1865; Russell, 1919; Poincaré, 1902; Blanché, 1975). As Bertrand Russell says:

«*The mere fact that something has happened a number of times makes both animals and humans wait for their return. And it is quite certain that our instincts cause us to believe that the sun will rise tomorrow, but perhaps we are not in a better position than the chicken whose neck has been twisted without its expecting it*» (Russell, 1989, cited in Vidal, 2005, p.211).

Several authors (Bernard, Carnap, Hume, Popper, Poincaré, Arnaud and Nicole, Russel etc.) have mentioned the epistemological problems of induction for different sciences. In the empirical sciences and humanities, these problems manifest themselves in questions relating to experimental method, the nature and *justification* of inductive *procedures* and their *validity*.

In mathematics, the problem of induction is not posed in the same way: it concerns the relations of ideas and not facts, as in the physical and natural sciences (Poincaré, 1902).

3. Typology of induction

3.1 Division of sciences

The sciences have remained grouped into a number of large entities, according to a certain commonality of objects and methods. Blanché (1972) mentions the existence of two groups of sciences. On the one hand, the formal sciences (logic and mathematics) are always exact and demonstrative, but abstract, intellectual and

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arbitrary. On the other hand, there are the sciences of reality (physics, biology, etc.), some of which are abstract and general, with the aim of discovering the laws governing the various classes of phenomena, while the others are concrete, particular and descriptive.

The philosopher Spencer (1872) came up with a classification closer to the modern conception of science:

-Abstract sciences: which deal with the general forms of phenomena (logic and mathematics).

-Abstract-concrete sciences: study phenomena in their fundamental elements (mechanics, physics, chemistry, etc.).

-Concrete sciences: deal with phenomena as a whole (astronomy, geology, biology, psychology, sociology, etc.).

This classification or the distinction between sciences makes induction unclear and vague, although it is recognized as a very fertile concept in science (indispensable tool that makes us go from finite to infinite), induction opens a great debate between interlocutors (philosophers, empiricists, semanticists, mathematicians etc.) of their justification procedures. Indeed, mathematical induction differs from both linguists and philosophers (Poincaré, 1902; Djabbar, 2018).

3.2. Real sciences (empirical, human and social)

Several authors distinguish between inductions in different fields. There are two types of induction: *complete* and *incomplete*.

For the empirical sciences (physics, chemistry, astronomy, medicine, etc.), the induction used is incomplete, as it is a scientific approach based on three stages: Observation, hypothesis and experiment. Similarly, for the social sciences, e.g. sociology, the induction used is incomplete, depending on certain hypotheses given in the form of interpretations and observations to analyze historical data, studies and statistical information.

In philosophy induction has a meaning related to the theory of knowledge or demonstration. The example of Aristotle who considers that induction is the *passage of particular cases to the universal which organizes them* can be cited.

In general, this induction is based on observations, regularities, analyses and experiments, and it takes time to find effective results or solutions to global problems such as epidemics (Corona virus), inflation in economics, their causes and treatments, and the prosperity of the digital society and its drawbacks.

3.3. Formal sciences (Mathematics)

In mathematics, the induction used corresponds to complete (*totalizing)* induction (or by Mathematical induction) (Poincaré, Russel, Djabbar, Vidal etc.). Before the formalization of reasoning by Mathematical induction, incomplete induction was used as a means of demonstration, leading some mathematicians to draw erroneous conclusions. The study illustrate some examples of erroneous incomplete induction:

Example 1: Euler (1780) studies the terms of the sequence (f_n) n defined on IN by:
$f_n = n^2 – n + 41$ depending on whether they are prime or not, $f_{41} = 41^2$ is not prime. The test for n=41 refutes the idea that for all natural integers n, $n^2–n+41$ is prime

Example 2: false induction: Fermat (1657) considers that the terms of the sequence (F_n) n defined on IN by: $F_n = 2^{2^n}+1$ are prime according to an incomplete induction. This induction leads to a false result, as Euler proved a century later, in 1732. Indeed, the sixth term for n=5: $F_n=2^{32} + 1$ is divisible by 641, whereas Fermat considers all terms to be prime.

It should also be noted that induction was used as a method of discovery. The study thus illustrate the terms of Aristotle who was already aware of it:

«*Induction is not a demonstration, but it shows something. We learn only by induction*

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or demonstration. But the demonstration is made from universal principles, and the induction, from particular cases. But it is impossible to acquire knowledge of the universes other than by induction» (Aristotle, c. 350 ACN, cited in Vidal, 2005, p. 192).

Nevertheless, these two examples show just how difficult it is in arithmetic to obtain a complete (or totalizing) induction, since it must, as in this case, often encompass infinity. This is the property of reasoning by Mathematical induction that Poincaré emphasizes in his famous passage on this reasoning in Science and Hypothesis.

The inductive method is a heuristic method aiming to arrive at a given generality in the form of conjectures describing a finite series of examples (Ernest, 1984).

Mathematical induction:

The statement of this principle in Logico-mathematical formalism is as follows:

$[P(n_0) \text{ and } (\forall n \geq n_0), (P(n) \Rightarrow P(n+1))] \Rightarrow (\forall n \geq n_0), P(n)$.

Égré (2015) makes a historical and epistemological study of Mathematical induction and shows that the justifications given by Poincaré, Frege and the formalists (for example Hilbert) clearly differ from each other; This reflects the difficulty of reducing it to a self-evident principle. Indeed, the foundation of induction in Poincaré is an act of the mind, which condenses infinity of logical inferences. Induction follows an explicit definition of the notion of natural whole number in Frege. Finally, the question concerning the basis of mathematical induction is a false problem: induction is just another axiom.

The inductive method is a heuristic method aiming to arrive at a given generality in the form of conjectures describing a finite series of examples (Ernest, 1984).

Ernest asserts that the mathematical induction method is indeed a rigorous form of deductive proof. To avoid this confusion, Ernest (1984) suggests that the difference between the heuristic inductive method and the method of proof by Mathematical induction should be carefully explained and that the former method should be called the generalization method (or at least appears under a name other than inductive reasoning).

3.4. Induction in mathematics didactics

In mathematics didactics, inductive reasoning takes on different meanings: Pólya (1958) characterizes induction as a particular way and combination of reasoning that leads to the discovery of general laws from the observation of examples.

According to Pedemonte (2002), induction is an ampliative inference that leads to the construction of new knowledge from the observation of particular cases that are generalized to a larger set of cases. . Pedemonte (2002) and Cabassut (2005) both use one of the characterizations of Peirce to define the inductive step as a reasoning of the form data-result therefore rule. According to Grenier (2012), induction is considered a generalization established on the basis of several particular cases. For Jeannotte (2015), inductive reasoning comes into play when we seek to infer a rule from data and observed regularities. She mentions that inductive reasoning plays a role in regularity generalization. According to Duquesne (2003): « *Inductive inference is based on observed regularities from which more general conclusions can be drawn*». So, it's a question of moving from the particular to the general.

***Learners' lack of understanding and difficulties with the concept of Mathematical induction*:**

Grenier (2012) stresses the need to clarify the concept of mathematical induction in order to understand the meaning of this principle, and to discuss it at the didactic level: The quantifiers «it exists» and «for everything» are indispensable for understanding the meaning of this principle ; they are often implicit in teaching, and sometimes replaced by inadequate formulations. Implication must be understood in the sense of mathematical logic. For example, $(P(n) \Rightarrow P(n+1))$ can be true for values of n for which P(n) is false. In other words, a property can be hereditary from a certain rank, and yet never be true. Initialization" verification is therefore necessary. The mathematical induction principle's relationship with infinity is often a source of difficulty. The complexity arises from the fact that we are trying to demonstrate a property relating to a finite number of elements, but only by studying a generic value n. In fact, this relationship with infinity is not the main obstacle to understanding the principle. It can be approached differently.

Mathematical induction is not taught as a concept. Indeed, it seems to be limited to an algebraic and mechanical manipulation of steps (Harel, 2001); it thus becomes a technique for obtaining general results from a number of particular cases (Harel, 2001; Dogan, 2016) or a poorly understood proof technique (Grenier; 2012; Gardes, Gardes and Grenier, 2016) or an algorithm (Soltani, 2019& 2023).

In mathematics, has the dual specificity of allowing the construction of objects and being a founding proof tool for many results in discrete mathematics (Grenier, 2012). In teaching, the concept of mathematical induction is little used, often misunderstood (Soltani, 2019& 2023), partly because it requires some mastery of mathematical logic. However, a great deal of didactic work has shown the importance of effectively incorporating the concepts of logic into teaching. Some of these works have focused on the notion of involvement (Fabert & Grenier, 2011), or implicit quantification in implicative propositions (Chellougui, 2009). Other studies have shown the importance of students and students building a precise logical language (Chellougui, 2003 & 2009; Mesnil 2014).

The didactic analysis of our Master's research (Soltani, 2019) highlighted the complexity of teaching and learning reasoning by mathematical induction in Arithmetic. In fact, mastery of logical notions, in particular implication, quantification and disjunction, is essential for a better understanding of this type of reasoning. Thus this reasoning taught as an algorithm generates some difficulties in students. These difficulties are identified in the mistakes made by these students in a mathematical proof. Syntactic errors include the omission of the implication and the quantifier in the heredity step. In addition, there is a misuse of the implication or quantifier at this stage. And semantic errors: assigning the wrong meaning to equivalence symbols and the universal quantifier. For example, the explanation of the initialization check makes no sense. Finally, the mixed type errors: succession of sentences with no logical link when justifying the heredity step, error in translating the $\Longrightarrow$ symbol to the$\Leftrightarrow$ symbol and incorrect use of the quantifier which assigns an incorrect meaning to the expression

4. Mathematical induction and recursion: interaction between mathematics and computer science

4.1 Definitions

The integration of mathematical concepts in particular mathematical and recursion in computer science teaching sheds light on the link between mathematical induction and recursion. The authors show some examples of applications in logic, computer science and mathematics that may be of interest to teachers to teach mathematical induction reasoning.

Mathematical induction: this is a type of reasoning that is applied to prove the properties of natural integers. A classic statement is as follows:

IF [there exists a natural number n_0 such that "p (n_0) is true" AND "for any natural number $n \geq n_0$, p(n) implies p(n+1) is true"] THEN [for any natural number n $\geq n_0$, p(n) is true].

In this reasoning, the logical connector's implication and conjunction are used in natural language, as well as the universal quantifier.

The statement of this reasoning in the logical-mathematical formalism is as follows:
$[P(n_0) \text{ and } (\forall n \geq n_0), (P(n) \Rightarrow P(n+1))] \Rightarrow (\forall n \geq n_0), P(n)$.

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Recursion: recursion is a concept that is frequently found in everyday life: stories within stories, films within films, paintings within paintings, and so on. In computing terms, an object is said to be recursive if it contains itself or is defined on the basis of itself.

The concept of recursion is especially emphasized in mathematical definitions. This is a very effective technique for solving problems quickly, simply and with fewer intermediate objects. However, the main problem is to determine the instruction (or instructions) that represent the breakpoint for the program to end in all cases.

Recursive algorithm: An algorithm or process is said to be recursive if it uses an iterative or recursive process to generate a result that may depend on p previous results. We then speak of a recursive algorithm or process of order p. Here is an example: The calculation of sequences is an example of recursive algorithms. For example, the Fibonacci sequence: For a natural number n≥2, $F_n = F_{n-1} + F_{n-2}$ is the basis of a recurrent algorithm of order 2.

4.2 Examples of recursion applications

Examples of applications are given below (Soltani, 2022):

Example 1: Calculation of the factorial of a positive integer n.

For a natural n "n-factorial"(written n!) is defined informally to be the product of all the natural numbers from 1 to n, inclusive: $n! = 1 \times 2 \times 3 \ldots \times n$.

The factorial of a natural number n can be represented as an equation: $n! = n \times (n-1)!$.

Algorithm:

Fact (n)
If n=0 then
| return 1
If not
| return n*fact (n-1)
End

This algorithm should give: fact(1)= 1*fact(0) =1*1=1, fact(2) = 2*fact(1)=2*1=2, fact(3) = 3*fact(2) =3*2=6 etc.

Example 2: Recognising the parity of a natural number n using knowledge of the parity of n-1.

To determine the parity of a natural number n using the parity of n-1, you can use the following equation: Parity (n) = 1- Parity (n-1). Here, Parity (n-1) represents whether n is even or odd.

Algorithm:

Given: a positive or null integer n
Result: returns true if n is even, false if not
If n=0 then
| return true
If not
| return is odd (n-1)
End

Example 3: Calculating the terms of a sequence:

A recursive sequence of order 1: Propose the algorithm of the TERME-N procedure which displays the first n terms of the sequence defined by: $U_0 = 5$ and for any natural number n, $U_{n+1} = 2U_n+1.5$.

The sequence can be represented by the equation: U(n)=2U(n-1)+1.5. Where U(0)=5 and n is a natural number.

Algorithm:

TERM-N procedure (n: integer)
Beginning
U =5
Write (U)
For i from 1 to (n-1), do
U = 2*U+1.5
Write (U)
End for
End

Fibonacci sequence: is a recursive sequence of order 2, each element of which obeys the following Mathematical induction relation: $F_0 =1$, $F_1 =1$ and for any integer n≥ 2, $F_n = F_{n-1} + F_{n-2}$. For a given integer n, calculate the nth term of the Fibonacci sequence. Write the Fibo functions (respectively iterative solution without table and recursive solution).

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The Fibonacci sequence can be defined recursively using the following equation: Fib(0) =1, Fib(1) = 1 and Fib(n) = Fib(n-1) +Fib(n-2).

Algorithm:

```
From the Fibo-recursive function
:( recursive solution)
Fibo function (n: integer): integer
Beginning
If n < =1 then
|  return 1
If not
|  return Fibo (n-1) + Fibo (n-2)
End if
End
```

4.3 Link between mathematical induction and recursion

Some authors have made the link between mathematical induction and recursion:

Leron and Zazkiz (1986) mentioned that prior mastery of recursion could facilitate learning proof by mathematical induction.

Anderson (1992) pointed out that the problems that can be solved recursion are those that have an inductive solution.

Polycarpou (2006) explores the correlation between understanding inductive definitions and mathematical induction proofs among computer science bachelor's students. The results of this study showed, on the one hand, that students who had a better understanding of inductive structural definitions, obtained better results in the application of mathematical induction evidence, and on the other hand that those who understood less inductive definitions tended to mechanically apply mathematical induction reasoning

Finally, Léon and Modeste (2020) present the concept of structural induction, which sheds light on the link between mathematical induction and recursion. They show a few examples of applications in logic, computer science and mathematics, which may be of interest to teachers responsible for teaching reasoning by Mathematical induction or, in the future, recursion.

Then computer science, specifically recursion using a recurrent algorithm can also play the role of confirmation or invalidation of incomplete induction because totalizing induction is difficult from a mental experiment because of its relationship with the infinite.

5. Discussion

The epistemological study of induction has finally enabled us to emphasize that each of the interlocutors in the induction debate has elucidated a difficult question, on its validity, justification, etc. It is useful to distinguish between the following concepts: incomplete and complete induction (by mathematical induction). This distinction makes inductive reasoning as a complex concept. Nevertheless, nowadays the incomplete induction is used in other scientific discipline (physics, biology, chemistry etc.) that is to say, a generalization established on the basis of particular cases. In mathematics, complete induction is used to demonstrate a generative result, but sometimes it is necessary to go through phases of incomplete induction. Formalising reasoning by mathematical as a complete form of inductive reasoning takes a long time before it is accepted by the mathematical community as a valid means of demonstration. The three philosophical answers or justifications to the problem of the foundation of mathematical induction, which proposed by Égré and by Boniface, «formalist», «intuitionist» and «logist» show the complexity of the understanding of this type of reasoning. . Although this difference between complete induction (mathematical induction) and incomplete induction, logist Bernard drew a universally valid law from a number of observations and experiences, it proceeds well by a generalization which has some analogy with Mathematical induction. The typology that each science gives for induction is not decisive, but reasoning is recognized as a very fertile concept in the sciences. . In general, this induction is based on observations, regularities, analyses and experiments, it takes time to find effective results or solutions to global problems such as the epidemic (corona virus), inflation in economics, their causes and treatments and the prosperity of the digital society and their disadvantages.

Formalizing reasoning by mathematical induction as a complete form of inductive reasoning takes an enormous amount of time before it is accepted by the mathematical community as a valid means of demonstration. A number of mathematics didactics studies have shown that learners have major cognitive difficulties with this type of reasoning. The authors have organised these difficulties into three categories: technical, mathematical and conceptual. Many factors are associated with these difficulties.

Inductive and recursion reasoning are now essential in education for their relevance and usefulness. The theory of recursion in computer science is practically the study of this reasoning in mathematics applied to recursive algorithms.

This study enables us to highlight the link between mathematical induction and recursion. This link fosters interdisciplinarity between computer science and mathematics to improve learners' understanding of these concepts.

6. Conclusion

Mathematical induction is a very difficult concept from an epistemological and cognitive point of view. In order to understand its meaning and overcome the epistemological obstacles and cognitive difficulties faced by learners, this article has proposed an obvious approach to the need to implement joint didactic situations involving notions of mathematical induction and recursion in computer science, for an effective and rigorous teaching of mathematical induction proofs.

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